Essential Question How can you find the surface area of a cone?
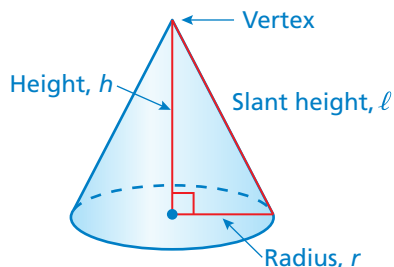
A cone is a solid with one circular base and one vertex.

Vertex

Height, h

Slant height, ℓ

Radius, r

1 ACTIVITY: Finding the Surface Area of a Cone

Work with a partner.

- Draw a circle with a radius of 3 inches.
- Mark the circumference of the circle into six equal parts.
- The circumference of the circle is $2(\pi)(3) = 6\pi$. So each of the six parts on the circle has a length of π. Label each part.
- Cut out one part as shown. Then, make a cone.

π π

3 in.

π π

π π

π

π π

π π
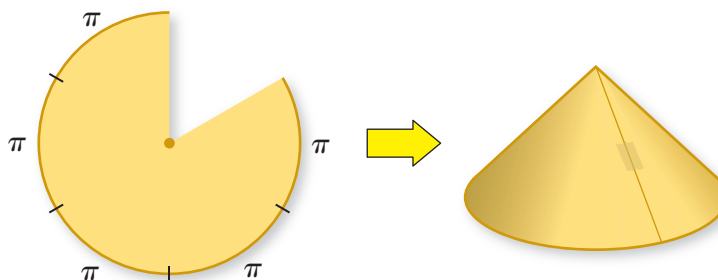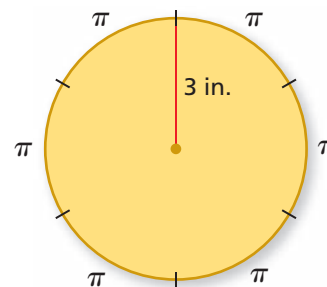
a. The base of the cone should be a circle. Explain why the circumference of the base is 5π.

b. Find the radius of the base.

c. What is the area of the original circle?

d. What is the area of the circle with one part missing?

e. Describe the surface area of the cone. Use your description to find the surface area, including the base.

## 2 ACTIVITY: Experimenting with Surface Area

Work with a partner.

- Cut out another part from the circle in Activity 1 and make a cone.
- Find the radius of the base and the surface area of the cone.
- Record your results in the table.
- Repeat this three times.
- Describe the pattern.

Shape					
Radius of Base					
Slant Height					
Surface Area					

3 ACTIVITY: Writing a Story

Write a story that uses real-life cones. Include a diagram and label the dimensions. In your story, explain why you would want to know the surface area of the cone. Then, estimate the surface area.
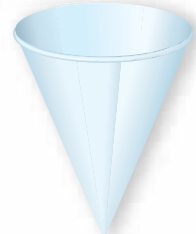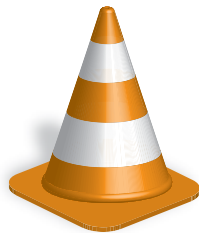
What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the surface area of a cone? Draw a diagram with your explanation.

Practice ➤ Use what you learned about the surface area of a cone to complete Exercises 4–6 on page 280.
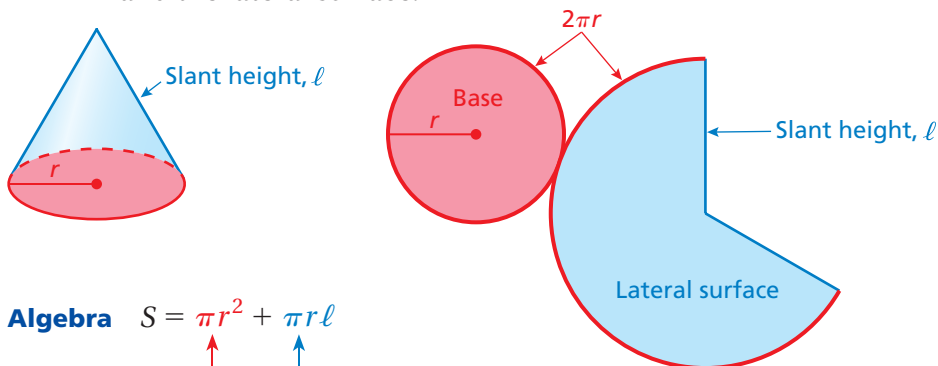
Check It Out
Lesson Tutorials
BigIdeasMath.com

The distance from the vertex of a cone to any point on the edge of its base is called the **slant height** of the cone.

Key Idea

Surface Area of a Cone

Words The surface area S of a cone is the sum of the areas of the base and the lateral surface.

Slant height, ℓ

$2\pi r$

Base

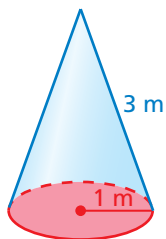r

Slant height, ℓ

Lateral surface

Algebra $S = \pi r^2 + \pi r \ell$
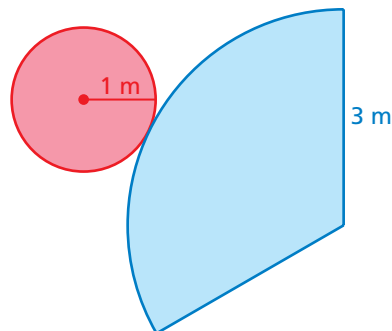
Area of base   Area of lateral surface

EXAMPLE 1 Finding the Surface Area of a Cone

Find the surface area of the cone. Round your answer to the nearest tenth.

3 m

1 m

Draw a net.

$$S = \pi r^2 + \pi r \ell$$
$$= \pi(1)^2 + \pi(1)(3)$$
$$= \pi + 3\pi$$
$$= 4\pi \approx 12.6$$

1 m

3 m

∴ The surface area is about 12.6 square meters.

On Your Own

Now You're Ready
Exercises 4–9

Find the surface area of the cone. Round your answer to the nearest tenth.

1.

2 ft   6 ft

2.

8 cm   4 cm

Multi-Language Glossary at BigIdeasMath.com.
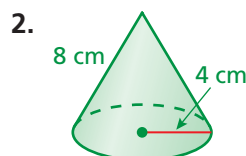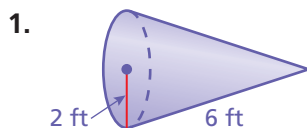
EXAMPLE **2** **Finding the Slant Height of a Cone**
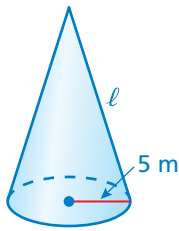
The surface area of the cone is 100π square meters. What is the slant height ℓ of the cone?

$$S = \pi r^2 + \pi r \ell \qquad \text{Write formula.}$$

$$100\pi = \pi(5)^2 + \pi(5)(\ell) \qquad \text{Substitute.}$$

$$100\pi = 25\pi + 5\pi\ell \qquad \text{Simplify.}$$

$$75\pi = 5\pi\ell \qquad \text{Subtract } 25\pi \text{ from each side.}$$

$$15 = \ell \qquad \text{Divide each side by } 5\pi.$$

∴ The slant height is 15 meters.

EXAMPLE **3** **Real-Life Application**

You design a party hat. You attach a piece of elastic along a diameter. (a) How long is the elastic? (b) How much paper do you need to make the hat?

a. To find the length of the elastic, find the diameter of the base.

$$C = \pi d \qquad \text{Write formula.}$$

$$22 \approx (3.14)d \qquad \text{Substitute.}$$

$$7.0 \approx d \qquad \text{Solve for } d.$$

∴ The elastic is about 7 inches long.

5 in.

$C = 22$ in.

b. To find how much paper you need, find the lateral surface area.

Remember

The diameter d of a circle is two times the radius r.

$$d = 2r$$

$$S = \pi r \ell \qquad \longleftarrow \qquad \boxed{\text{Do not include the area of the base in the formula.}}$$

$$= \pi(3.5)(5) \qquad \text{Substitute.}$$

$$= 17.5\pi \approx 55 \qquad \text{Multiply.}$$

∴ You need about 55 square inches of paper to make the hat.

🔴 **On Your Own**

**Now You're Ready**
Exercises 10–14

3. **WHAT IF?** In Example 2, the surface area is 50π square meters. What is the slant height of the cone?

4. **WHAT IF?** In Example 3, the slant height of the party hat is doubled. Does the amount of paper used double? Explain.

Vocabulary and Concept Check

1. **VOCABULARY** Is the base of a cone a polygon? Explain.

2. **CRITICAL THINKING** In the formula for the surface area of a cone, what does $\pi r \ell$ represent? What does πr^2 represent?

3. **REASONING** Write an inequality comparing the slant height ℓ and the radius r of a cone.
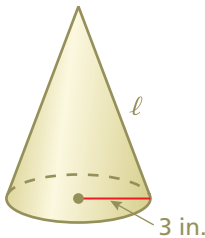
Practice and Problem Solving

Find the surface area of the cone. Round your answer to the nearest tenth.
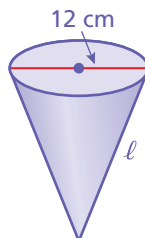
1 **4.** 6 in. / 3 in.

5. 5 m / 4 m

6. 9 mm / 5 mm

7. 10 ft / 7 ft

8. 5 cm / 11 cm

9. 8 yd / 12 yd
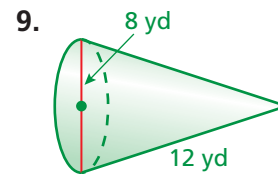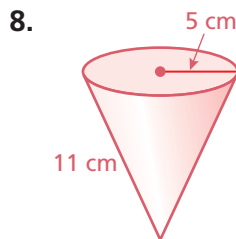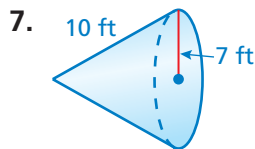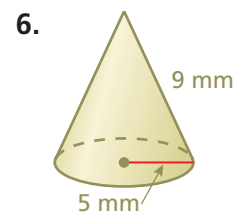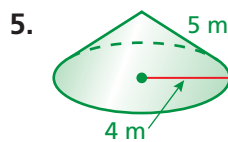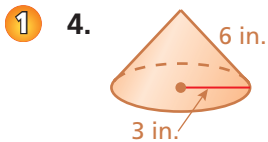
Find the slant height ℓ of the cone.

2 **10.** $S = 33\pi$ in.^{2}
ℓ / 3 in.

11. $S = 126\pi$ cm^2
12 cm / ℓ

12. $S = 60\pi$ ft^2
ℓ / 5 ft
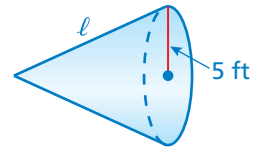
3 **13. NÓN LÁ** How much material is needed to make the Nón Lá Vietnamese leaf hat?

13 in.

20 in.

14. PAPER CUP A paper cup shaped like a cone has a diameter of 6 centimeters and a slant height of 7.5 centimeters. How much paper is needed to make the cup?
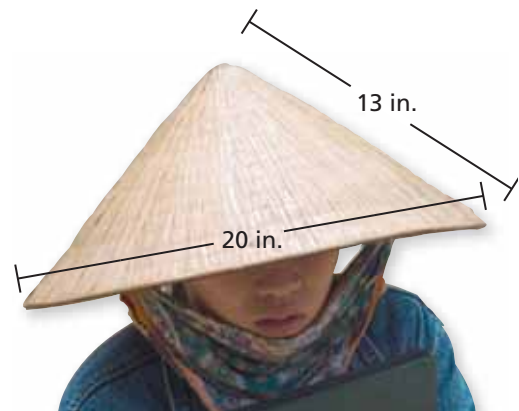
Find the surface area of the cone with diameter d and slant height ℓ.

15. $d = 2$ ft

$\ell = 18$ in.

16. $d = 12$ cm

$\ell = 85$ mm

17. $d = 4$ yd

$\ell = 10$ ft

18. ROOF A roof is shaped like a cone with a diameter of 12 feet. One bundle of shingles covers 32 square feet. How many bundles should you buy to cover the roof?

13 ft

19. MEGAPHONE Two stickers are placed on opposite sides of the megaphone. Estimate the percent of the surface area of the megaphone covered by the stickers. Round your answer to the nearest percent.

2.25 ft
6 in.
1.2 ft
6 in.
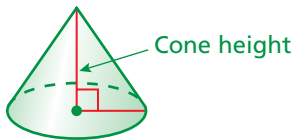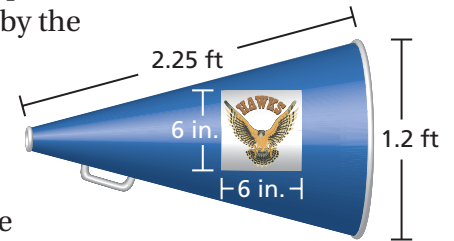
20. REASONING The height of a cone is the distance between the base and the vertex. Which is greater, the height of a cone or the slant height? Explain your reasoning.

Cone height

21. GEOMETRY The surface area of a cone is also given as $S = \frac{1}{2}C\ell + B$, where

C is the circumference and ℓ is the slant height. What does $\frac{1}{2}C\ell$ represent?

22. **Critical Thinking** A cone has a diameter of x millimeters and a slant height of y millimeters. A square pyramid has a base side length of x millimeters and a slant height of y millimeters. Which has the greater surface area? Explain.

Fair Game Review What you learned in previous grades & lessons

Find the area of the shaded region. Use 3.14 for π. *(Skills Review Handbook)*

23.

6 in.
4 in.
15 in.

24.

3 m
5 m

25.

4 ft
8 ft
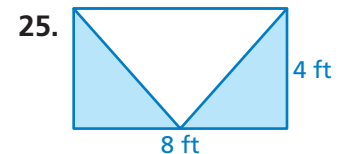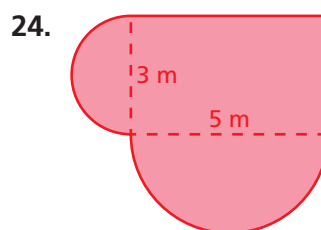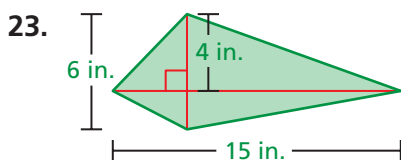
26. MULTIPLE CHOICE Which best describes a translation? *(Section 5.5)*

Ⓐ a flip

Ⓑ a slide

Ⓒ a turn

Ⓓ an enlargement